Cyber Security Analysis of Power Networks by Hypergraph Cut Algorithms

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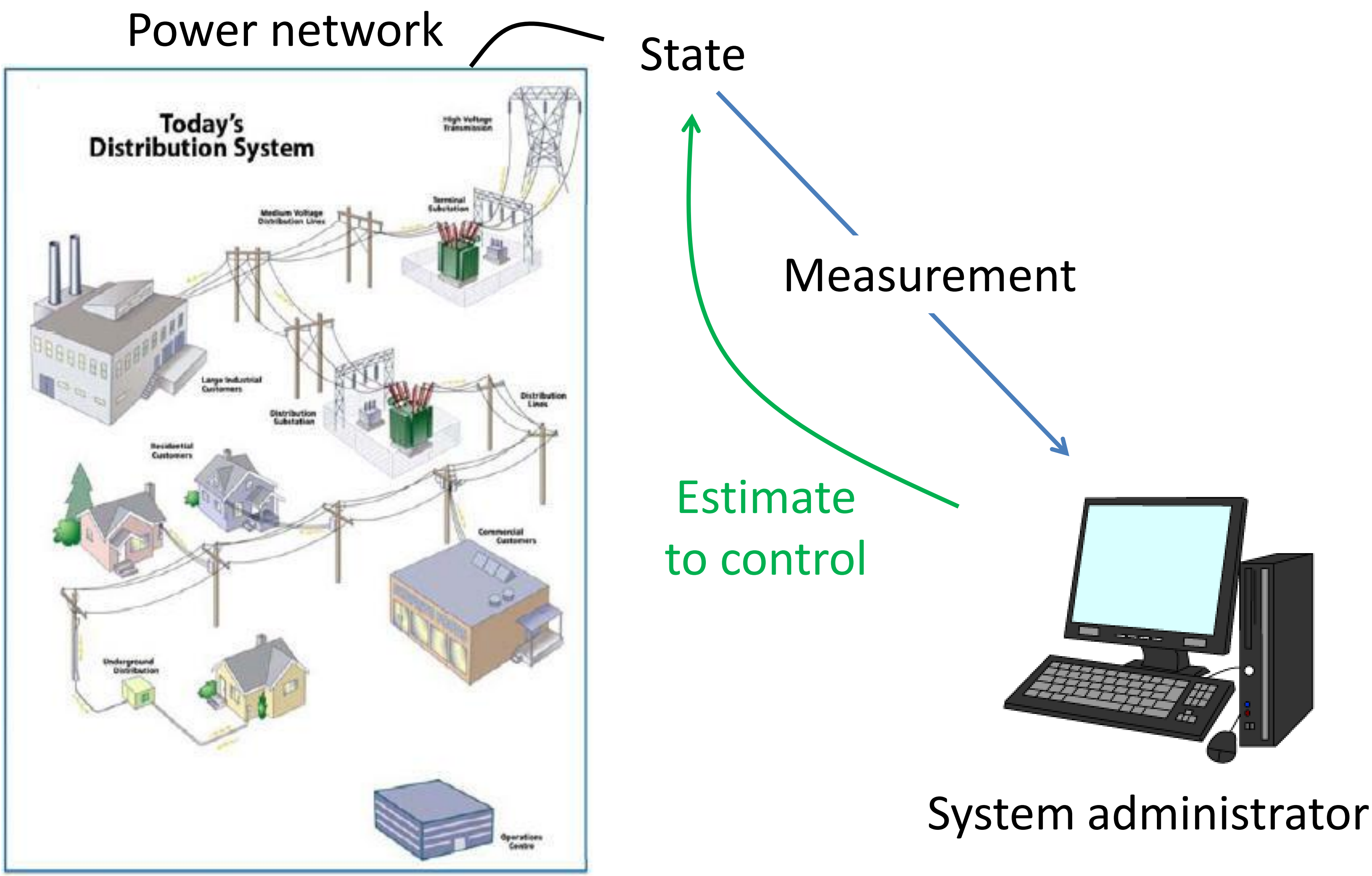
Cyber Threat to Power Networks



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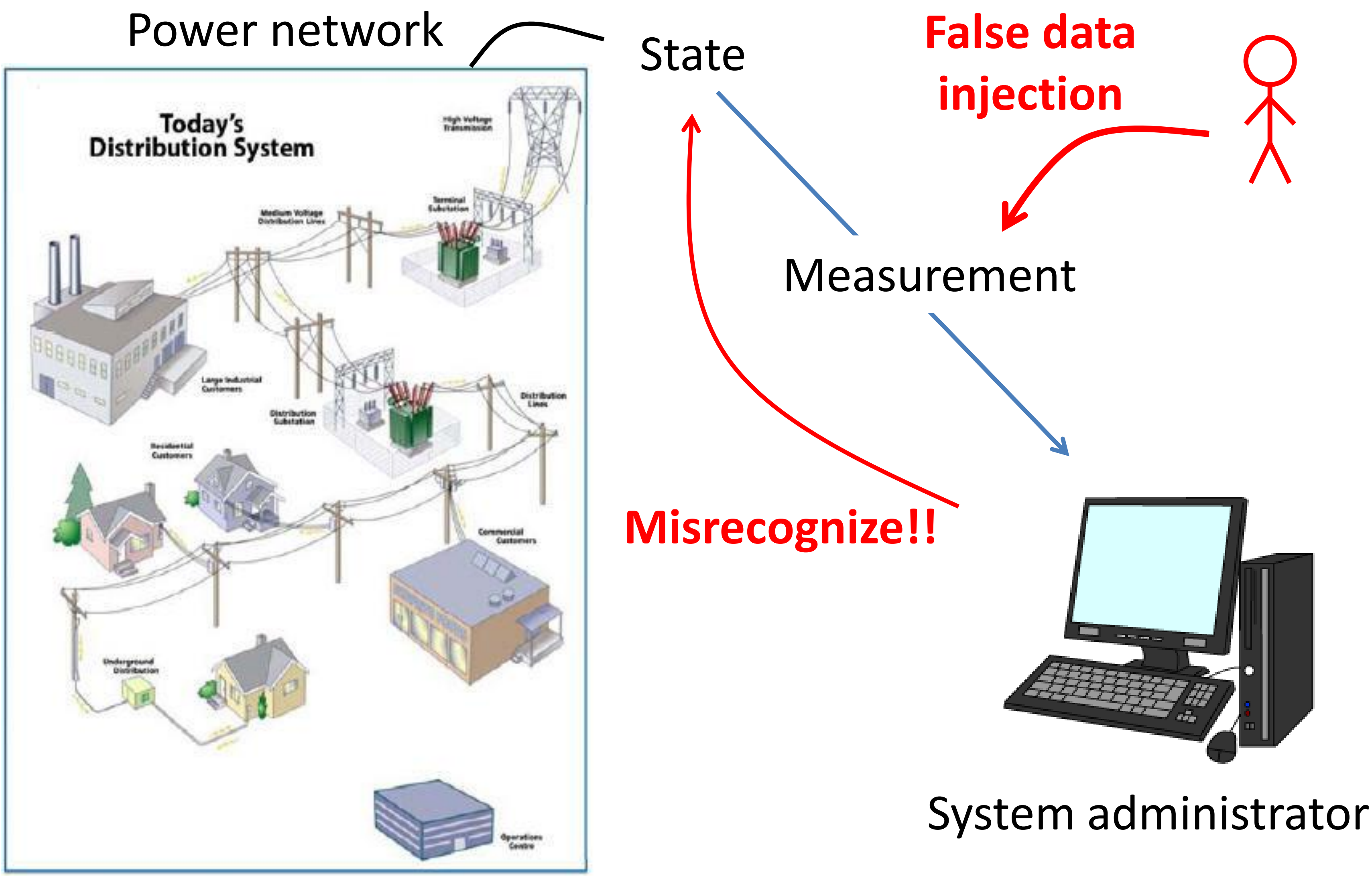

Outline

- Model and Problem Definitions
  - **Undetectable (false data injection) attacks**
  - **Sparsest attack** problem (**Global security** analysis)
  - **Security index** problem (**Local security** analysis)
- Existing Methods vs. **Proposed Methods**
  - Approx. by LP-relaxation
  - Approx. by min-cut in graphs
  - Exact by min-cut in auxiliary graphs
  - **Exact by min-cut in hypergraphs (Proposed)**
- Experimental Results

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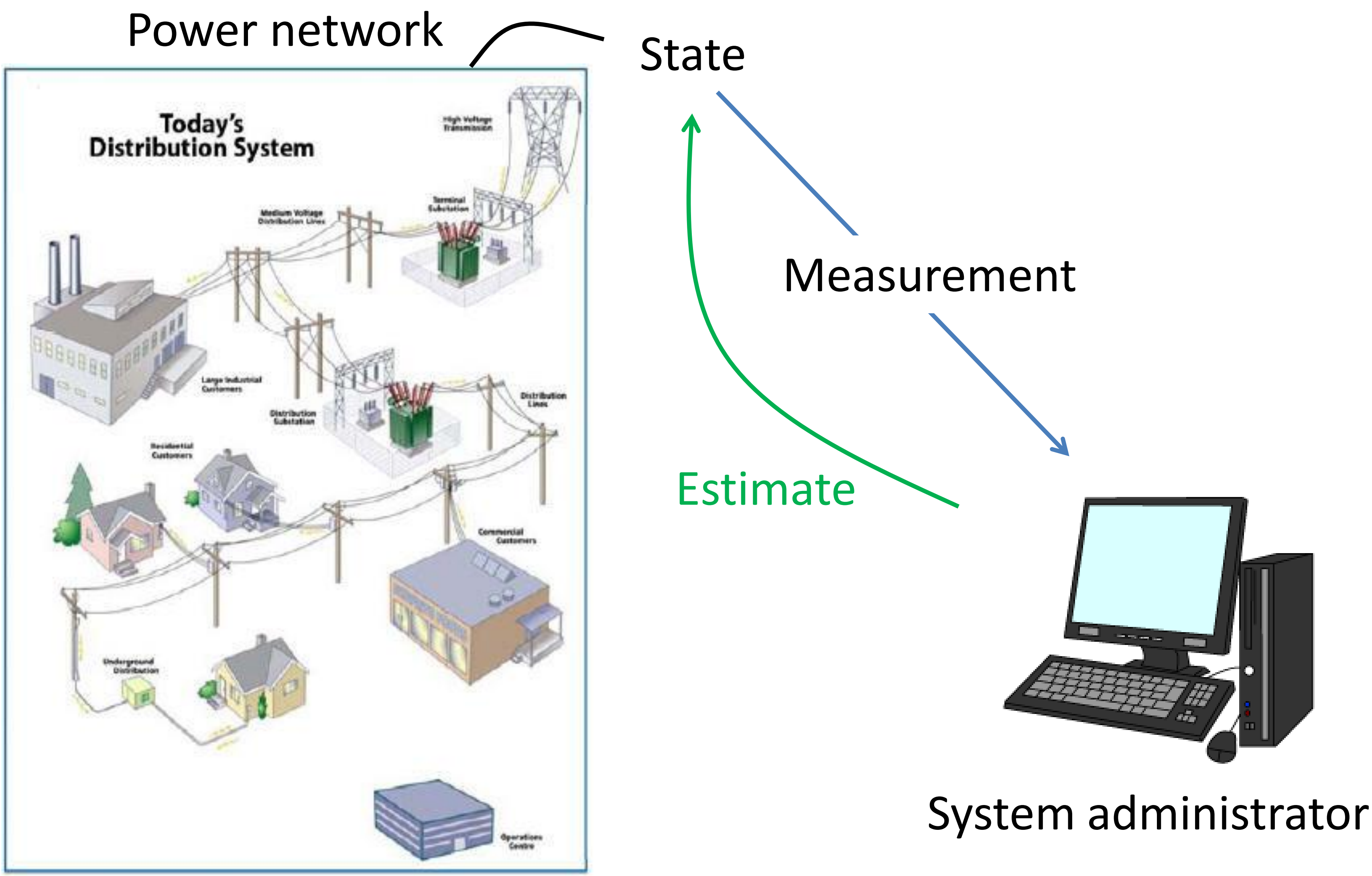
Linearized State Estimation Model



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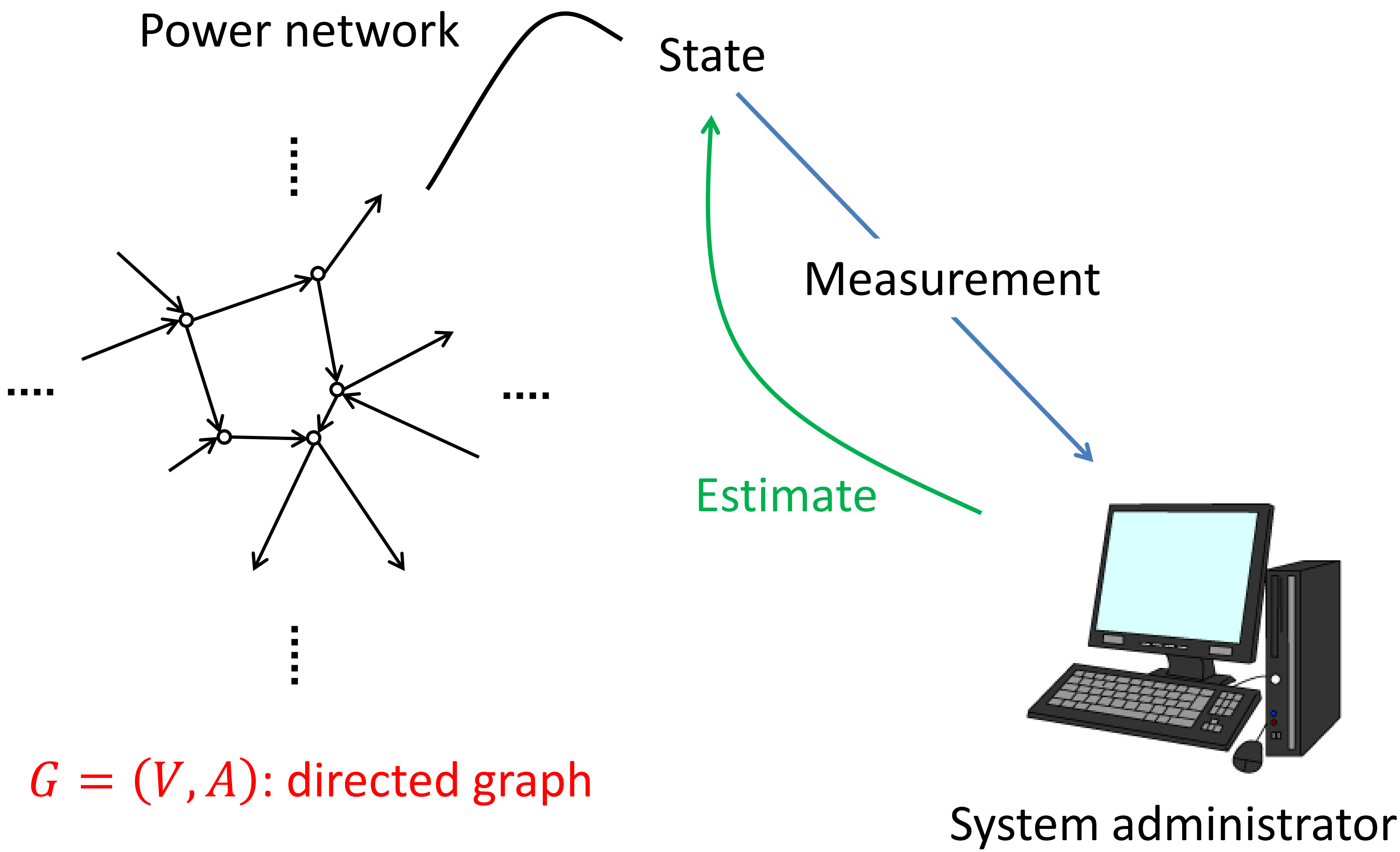
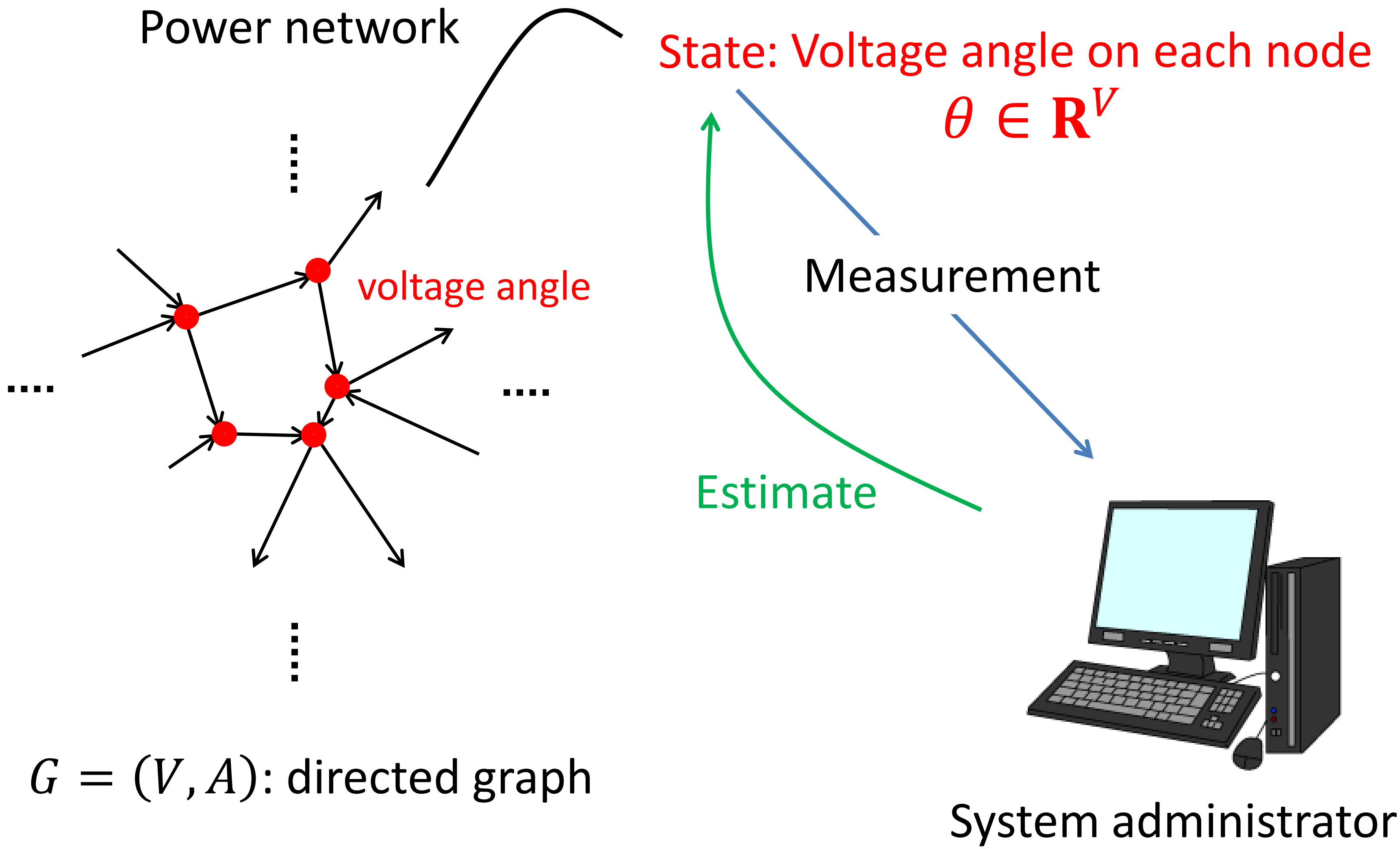


$G = (V, A)$: directed graph

Linearized State Estimation Model



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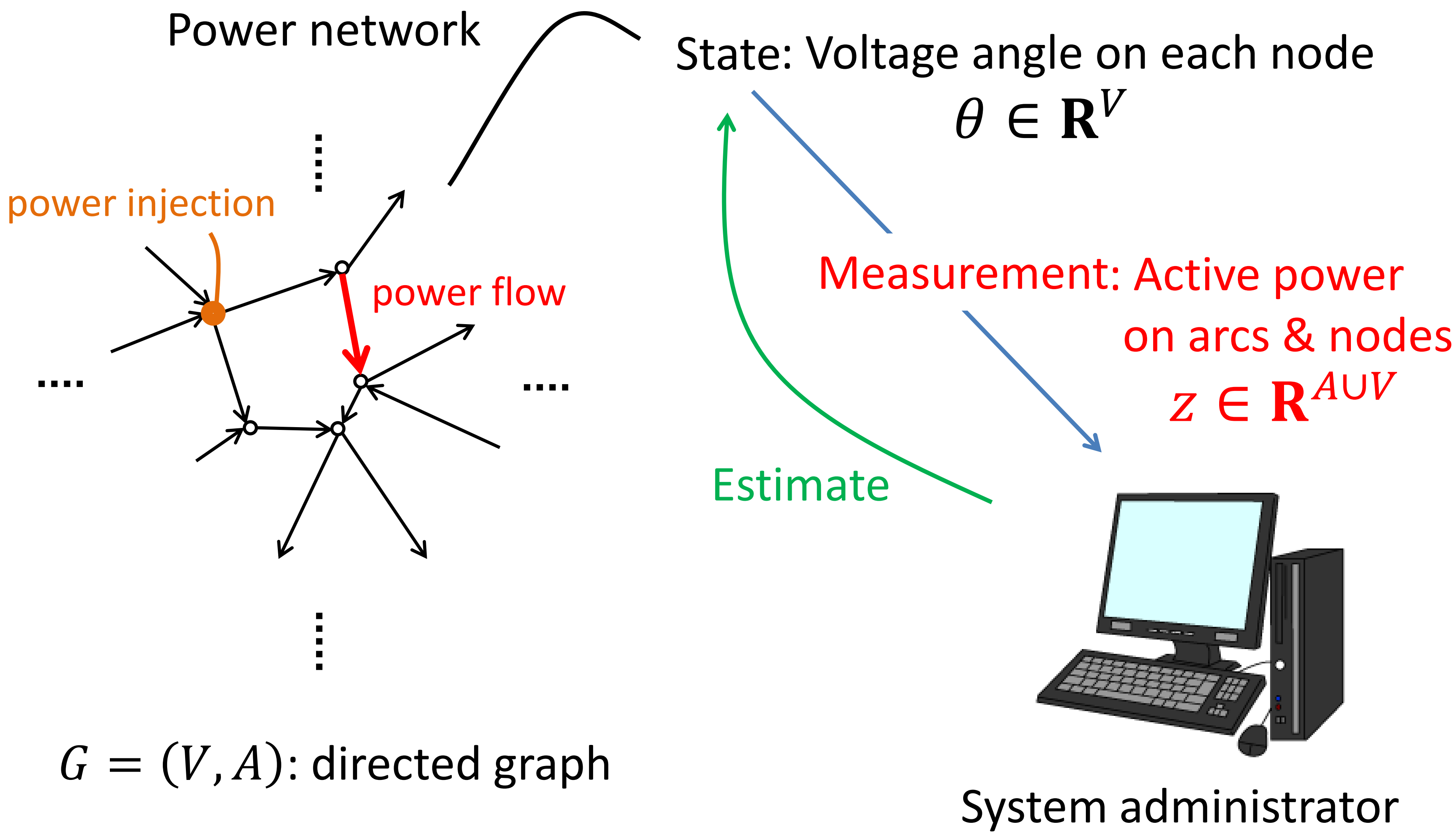
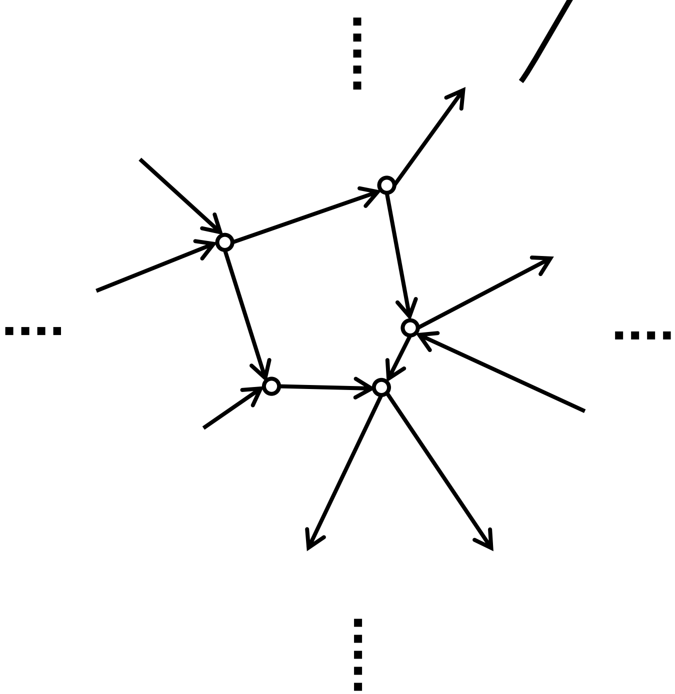


Linearized State Estimation Model

Power network



State: Voltage angle on each node

$$\theta \in \mathbf{R}^{V}$$

Measurement: Active power on arcs & nodes

$$z \in \mathbf{R}^{A \cup V}$$

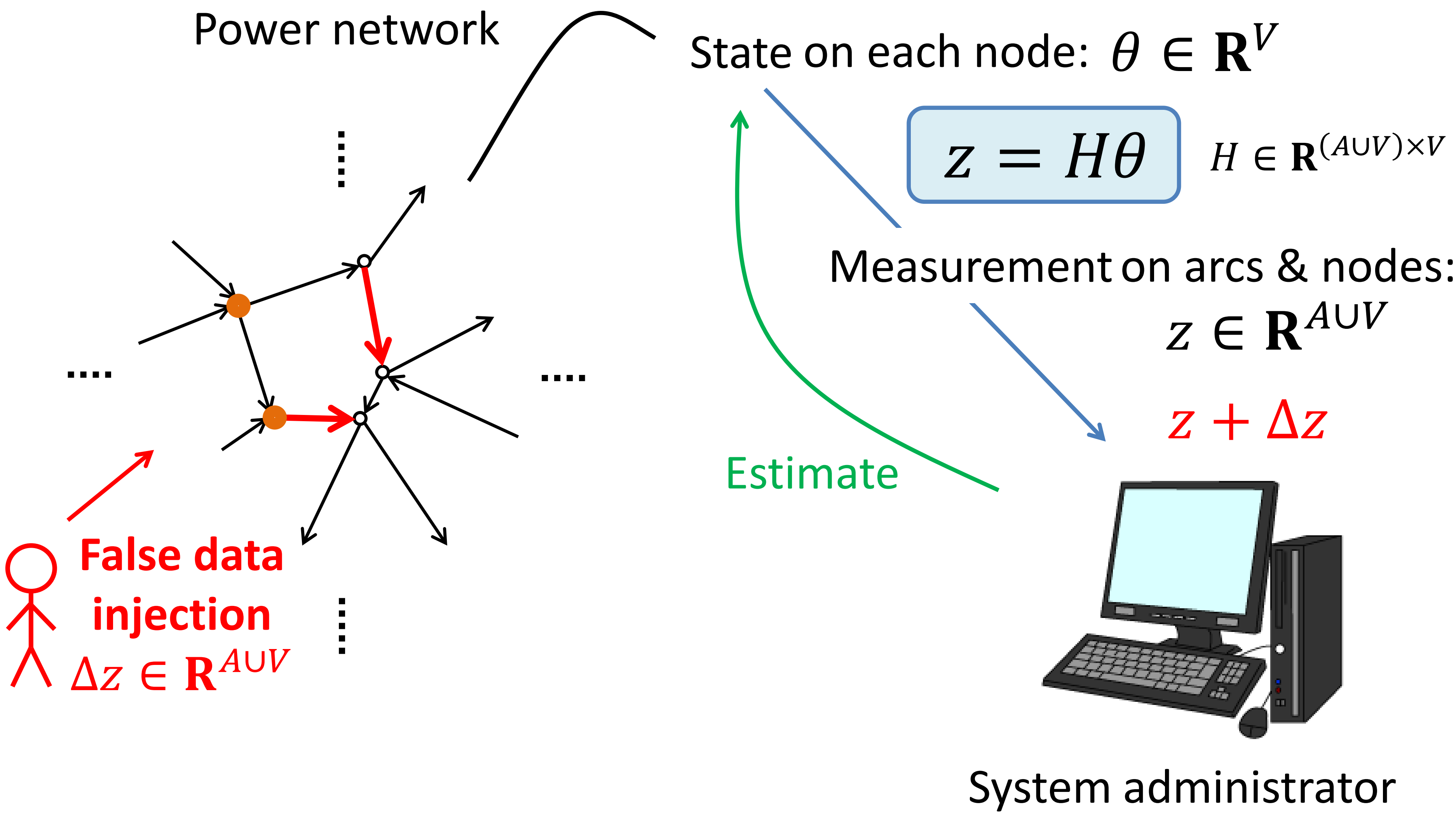
Estimate

$$z = H\theta$$

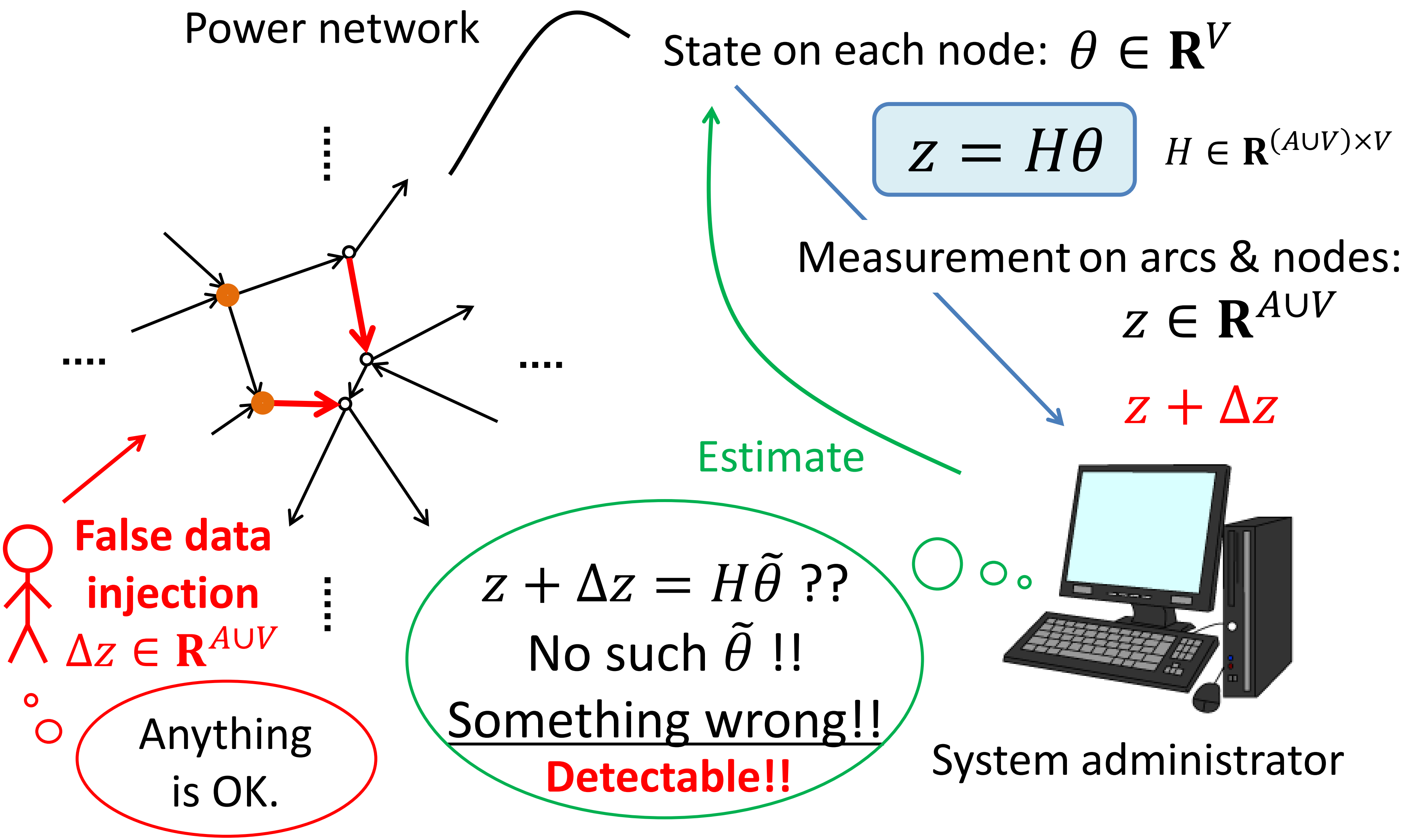
System administrator

$G = (V, A)$: directed graph $\quad H \in \mathbf{R}^{(A \cup V) \times V}$

False Data Injection



False Data Injection



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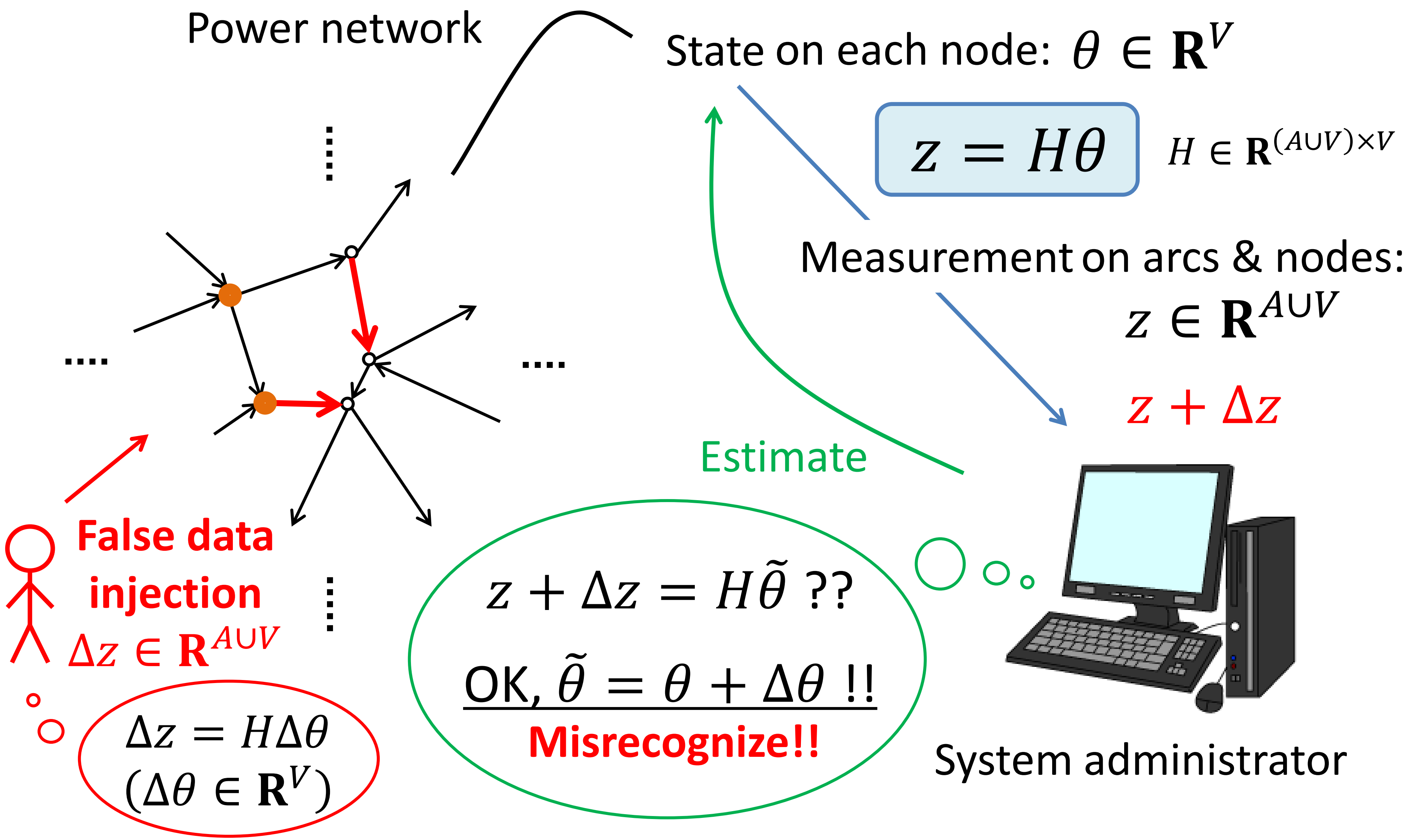


Undetectable (False Data Injection) **Attack**

(Liu, Ning, Reiter 2009)

A difference $\Delta z \in \mathbf{R}^{A \cup V}$ of measurement values is called an ***undetectable attack***.

$$\overset{\text{def}}{\Leftrightarrow} \ \exists \Delta\theta \in \mathbf{R}^{V} \text{ s.t. } \Delta z = H\Delta\theta$$

Actual: $z = H\theta$

Attack: $\Delta z = H\Delta\theta$

Misrecognition: $z + \Delta z = H(\theta + \Delta\theta)$

Sparsest Attack (Global Security)

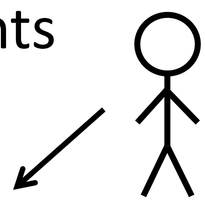
(Liu, Ning, Reiter 2009)

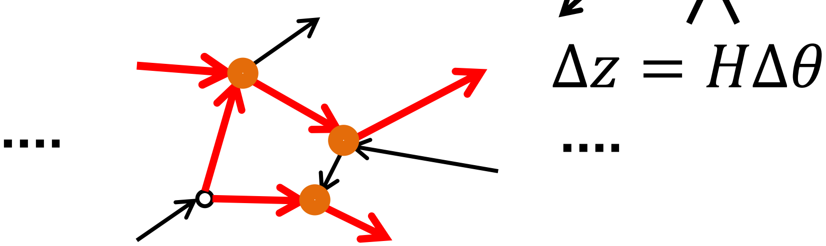
A nonzero **undetectable attack** $H\Delta\theta \in \mathbf{R}^{A \cup V} \setminus \{\mathbf{0}\}$ with the **fewest nonzero entries (attacked points)**

$$\begin{array}{ll} \underset{\Delta\theta \in \mathbf{R}^V}{\text{minimize}} & \|H\Delta\theta\|_0 \\ \text{subject to} & H\Delta\theta \neq \mathbf{0} \end{array}$$

Attacking **many** points

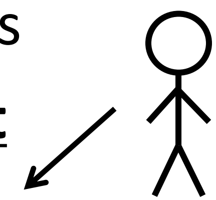
→ **Easy to prevent**

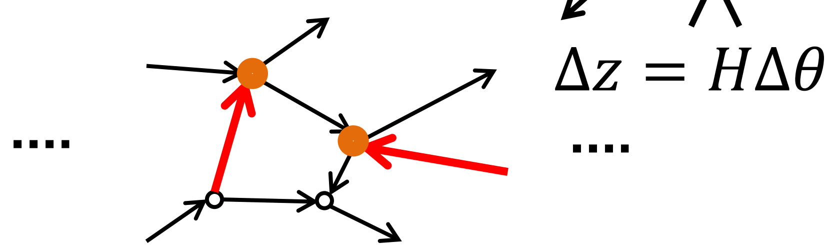




Attacking **few** points

→ **Hard to prevent**





Security Index (Local Security)

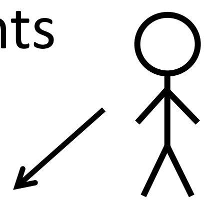
(Sandberg, Teixeira, Johansson 2010)

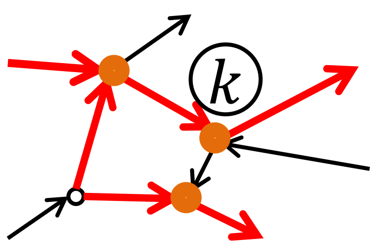
The **minimum number of nonzero entries** of an **undetectable attack** $H\Delta\theta \in \mathbf{R}^{A \cup V}$ **to attack a specified arc or node** $k \in A \cup V$

$$\underset{\Delta\theta \in \mathbf{R}^V}{\text{minimize}} \quad \|H\Delta\theta\|_0$$
$$\text{subject to} \quad H_k \Delta\theta \neq 0$$

Attacking **many** points

→ **Easy to prevent**





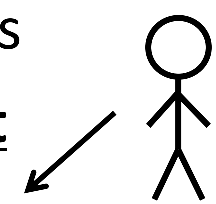
....

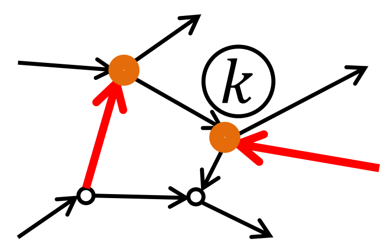
$\Delta z = H\Delta\theta$

....

Attacking **few** points

→ **Hard to prevent**





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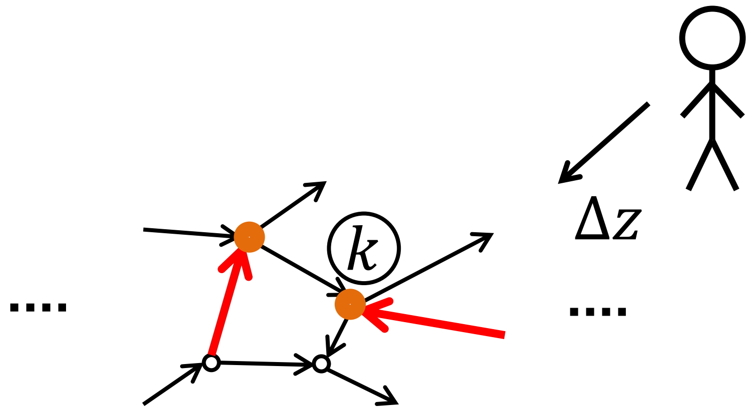
Sparsest Attack and Security Index

Fact

Any **sparsest attack** attains the **security indices** of the arcs and nodes to be attacked.



Δz is a **sparsest attack**.

$\Downarrow$

(**security index** of k) $= \|\Delta z\|_0$

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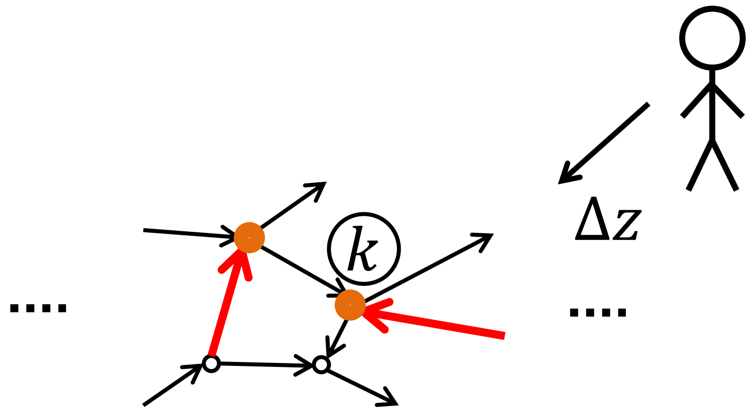

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(**security index** of k) $= \|\Delta z\|_0$

A **sparsest attack** can be found by computing the **security indices** of **ALL** arcs and nodes!

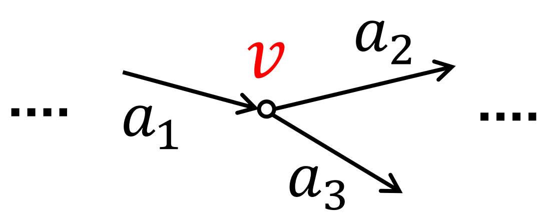
Sparsest Attack and Security Index

Fact

Any **sparsest attack** attains the **security indices** of the arcs and nodes to be attacked.

Fact

The **security index** of a node is equal to the **minimum** **security index** among its incident arcs'.



$$(\textbf{S.I.}\text{ of } v) = \min_{i=1,2,3} (\textbf{S.I.}\text{ of } a_i)$$

A **sparsest attack** can be found by computing the **security indices** of **ALL** arcs!!!

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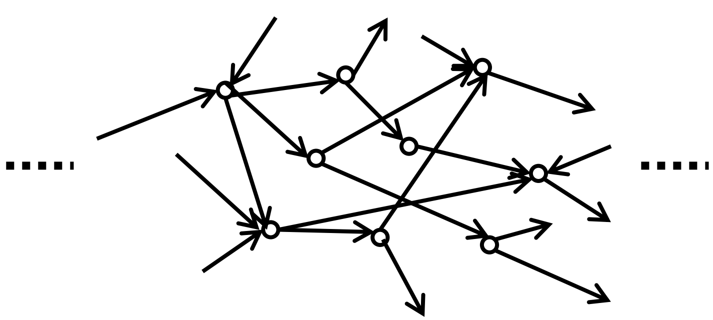
Solution Methods for **Security Index**

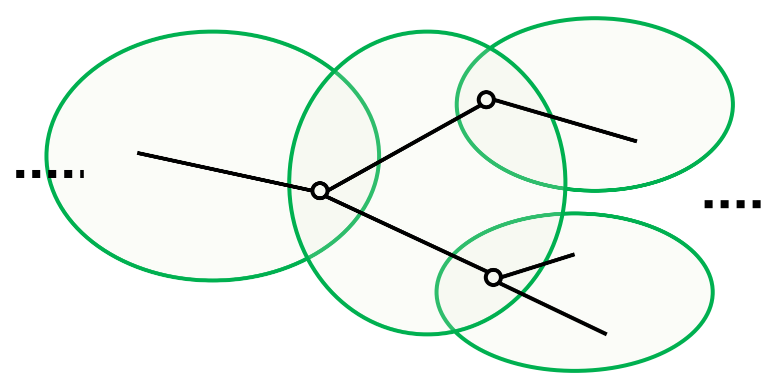
Approx. by min-cut
(Sou, Sandberg, Johansson 2011)

Approx. by LP-relax
(Sou, Sandberg, Johansson 2013)

Exact by min-cut
in auxiliary graph
(Hendrickx, Johansson, Junger, Sandberg, Sou 2012)

Exact by min-cut
in hypergraph





Solution Methods for Sparsest attack

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Exact by min-cut
in auxiliary graph
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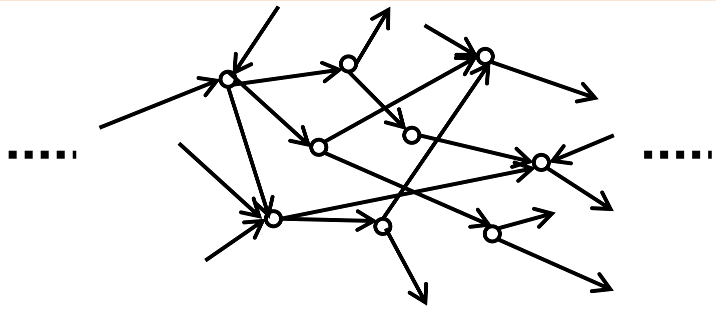
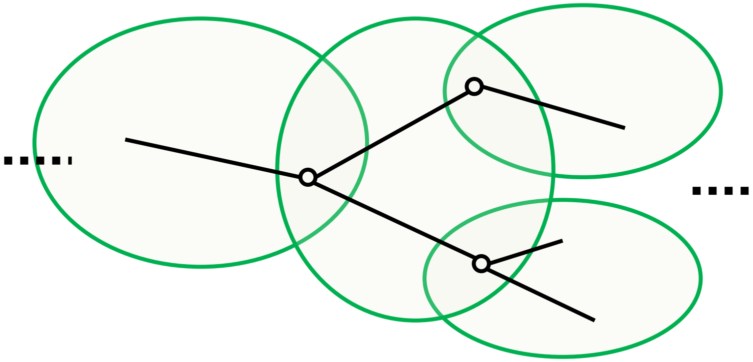
Security indices of ALL arcs

Exact by min-cut
in hypergraph

Single computation!!

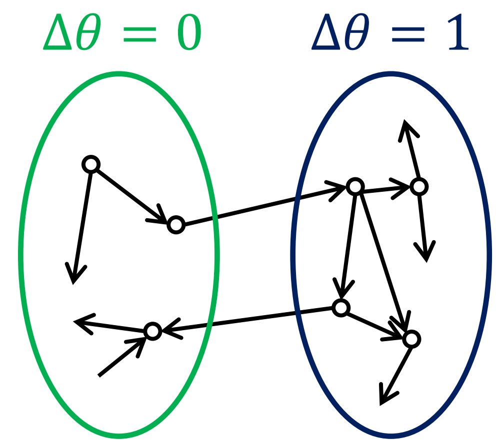




Why min-cut?

Elementary Attack

An undetectable attack $H\Delta\theta \in \mathbf{R}^{A \cup V}$ is ***elementary***. $\overset{\text{def}}{\Leftrightarrow}$ $\Delta\theta \in \{0, 1\}^V$



Lemma (Sou et al. 2011)

For any arc or node, there exists an **elementary attack** attaining the **security index**.

→ Consider only **elementary attacks**

→ Assign 0 or 1 to each node **(Bipartition the node set V)**

Elementary Attack

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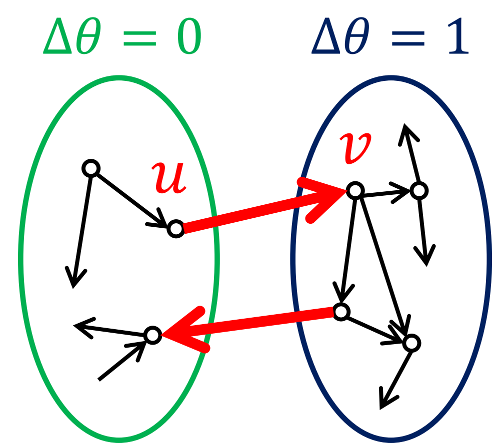

Fact

An arc $uv \in A$ **is attacked** in an **elementary attack**.

$\Leftrightarrow \Delta\theta(u) \neq \Delta\theta(v)$

$\Leftrightarrow uv$ **is cut off** by separating 0-nodes and 1-nodes.

→ # of **attacked arcs** = # of **arcs cut off** = **cut capacity**

→ Approx. by min-cut (Sou et al. 2011)

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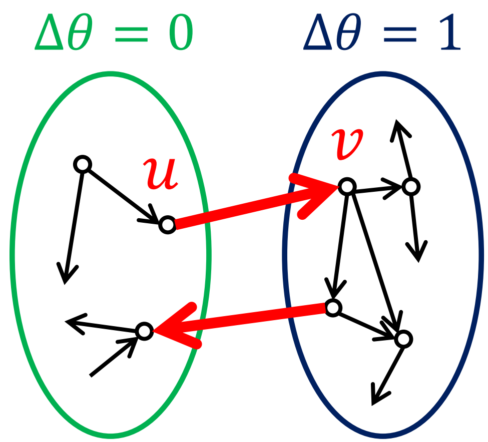

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→ # of **attacked arcs** = # of **arcs cut off** = **cut capacity**

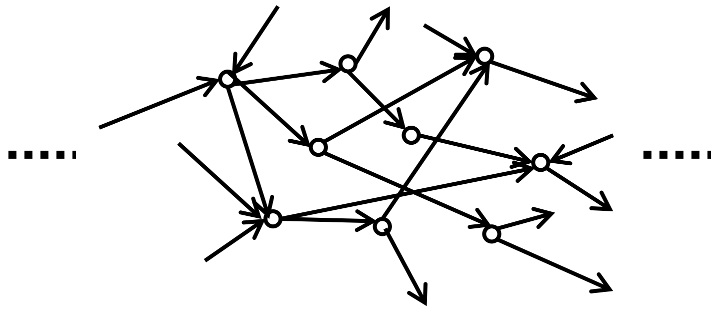
→ Approx. by min-cut (Sou et al. 2011) How about **attacked nodes**?

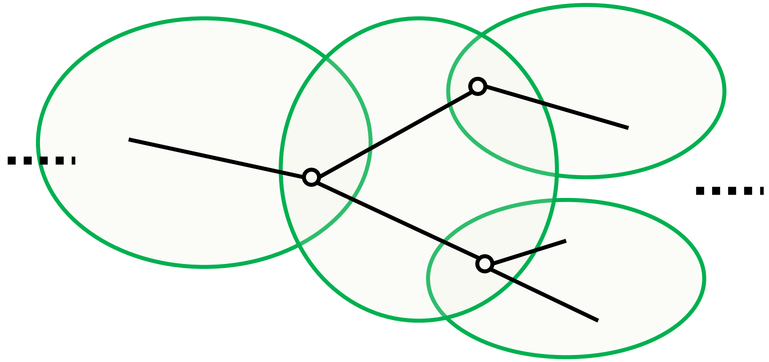
Counting **Attacked Nodes**

Construct auxiliary graph

(Hendrickx, Johansson, Junger, Sandberg, Sou 2012)

Use hypergraph





- Large size
- A **sparsest attack** requires **(# of arcs) min-cut comps.**

- No additional node
- A **sparsest attack** can be found by **single min-cut computation**!!

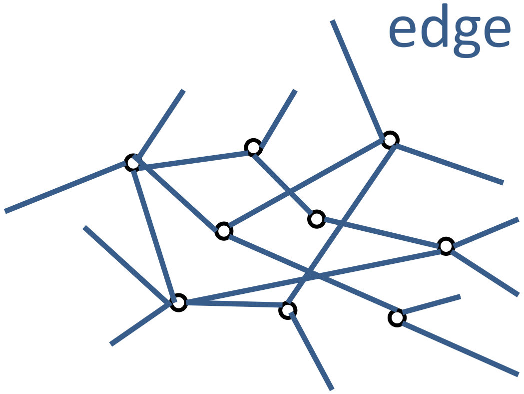
Hypergraphs

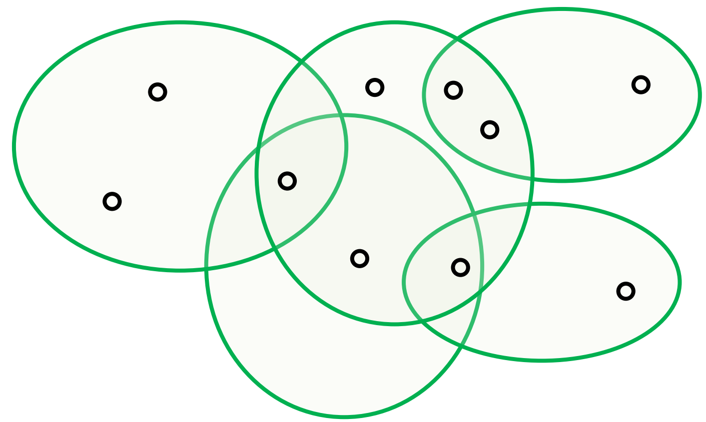
Undirected graph

Hypergraph

hyperedge

edge ⇔ hyperedge
of size 2



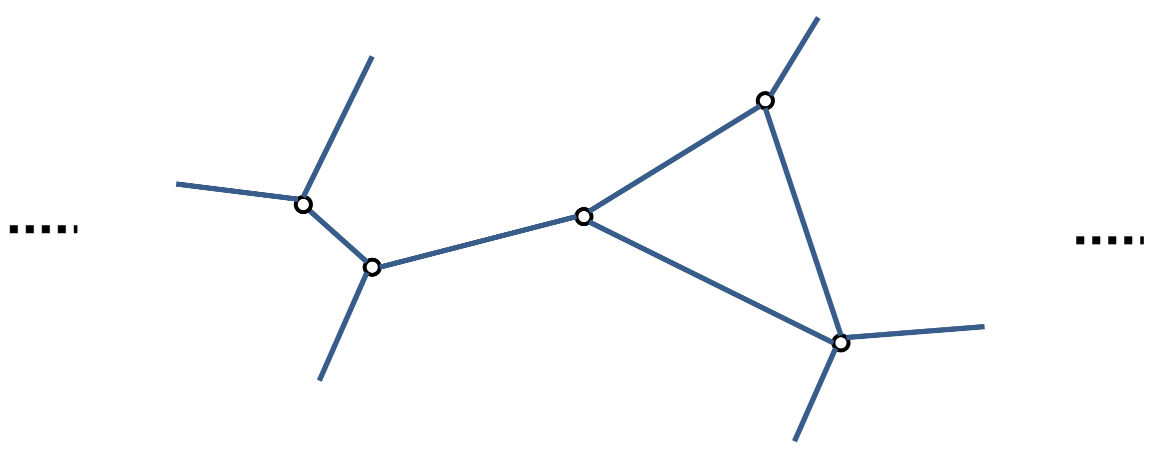


Each edge connects **two nodes**.

Each hyperedge connects **an arbitrary number of nodes**.

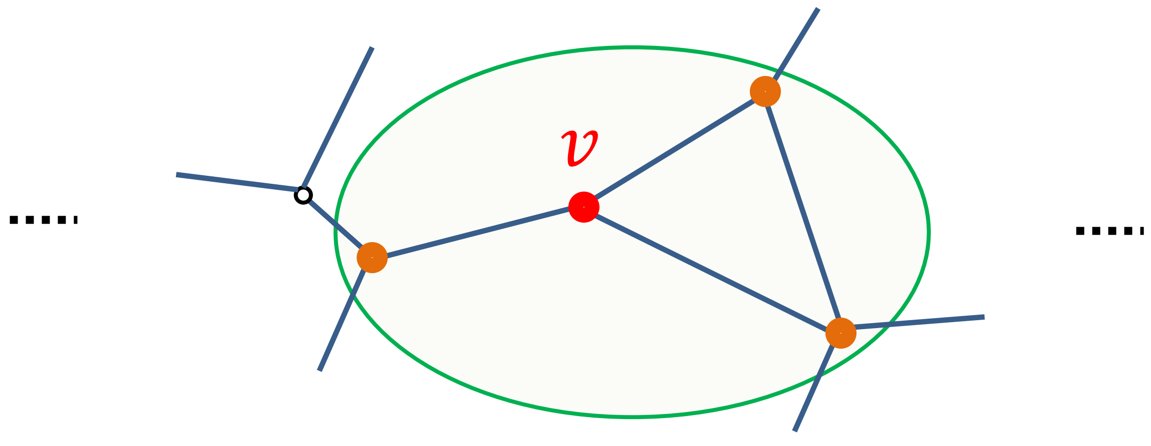
Construction of Hypergraph

- Start with the input graph (ignoring the direction)



Construction of Hypergraph

- Start with the input graph (ignoring the direction)
- For each node $v \in V$, add a **hyperedge** consisting of **the node v itself** and **all neighbors of v**.



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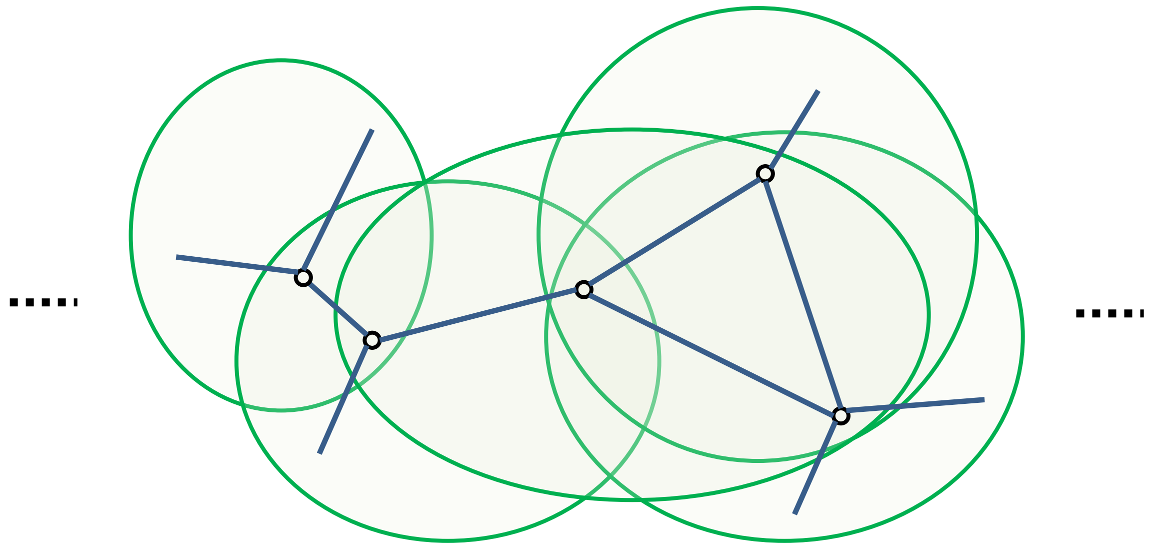


Construction of Hypergraph

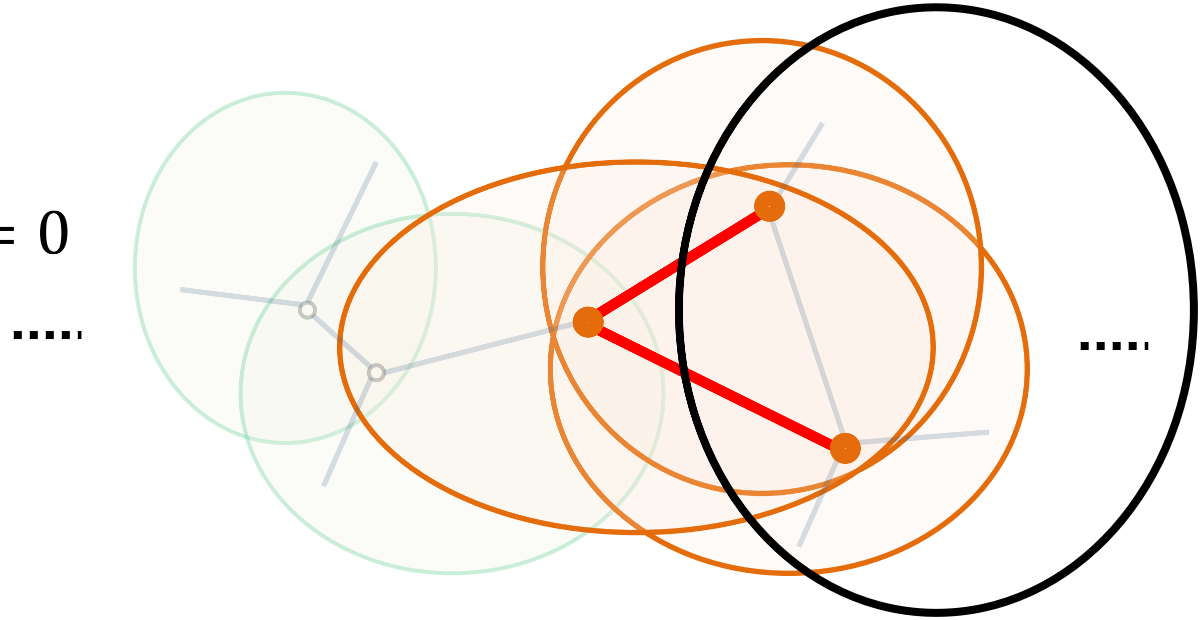
Lemma (Y.-O.-T.-I. 2014)

Cut capacity in this hypergraph

||

\# of **arcs & nodes to be attacked**

$\Delta\theta = 0$

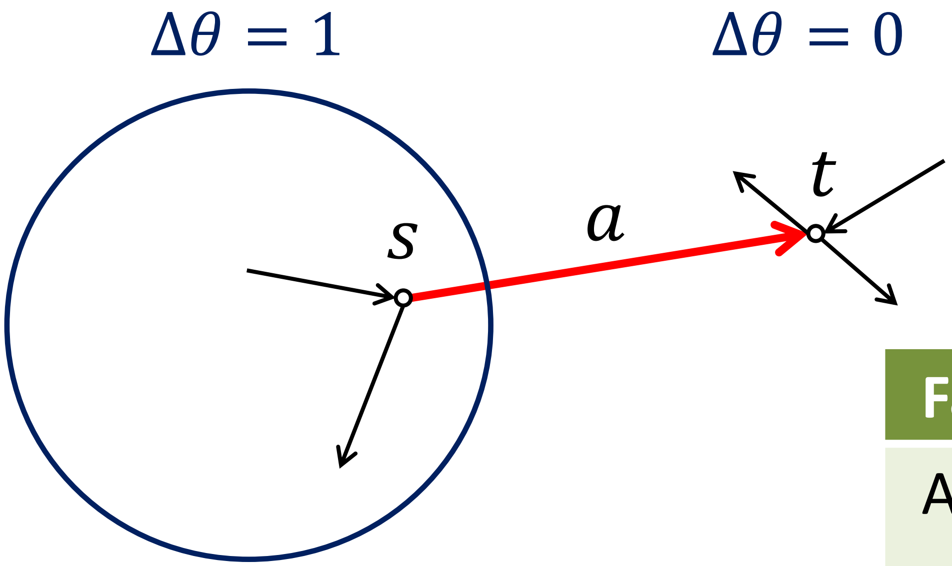


$\Delta\theta = 1$

Computing **Security Index**

Computing the **security index of an arc** $a = st \in A$

$\rightarrow$ Finding **a minimum s—t cut** in a hypergraph



Fact

An arc $st \in A$ **is attacked**.

$\Leftrightarrow \Delta\theta(s) \neq \Delta\theta(t)$

$\Leftrightarrow st$ **is cut off**.

Computing **Security Index**

Computing the **security index of an arc** $a = st \in A$

$\rightarrow$ Finding **a minimum** s—t **cut** in a hypergraph

Theorem (Y.-O.-T.-I. 2014)

For any arc in any directed graph $G = (V, A)$,
one can compute the **security index** in $\mathrm{O}(|V||A|)$ time.

- By a **hypergraph min** s—t **cut** algorithm (Pistorius, Minoux 2003)
- The same order as the existing exact method (Hendrickx et al. 2012), but **faster in practice** because their auxiliary graph is large.

Finding **Sparsest Attack**

Finding a **sparsest attack** in the whole network

$\rightarrow$ Finding **a minimum cut** in a hypergraph

Theorem (Y.-O.-T.-I. 2014)

For any directed graph $G = (V, A)$, one can find **a sparsest attack** in $\mathrm{O}(|V||A| + |V|^2 \log|V|)$ time.

- By a **hypergraph min-cut** algorithm (Klimmek, Wagner 1996)
- **Essential speeding up!!**

  Applying the existing exact method (Hendrickx et al. 2012) to all arcs

  $\rightarrow$ $\mathrm{O}(|V||A|^2)$ time

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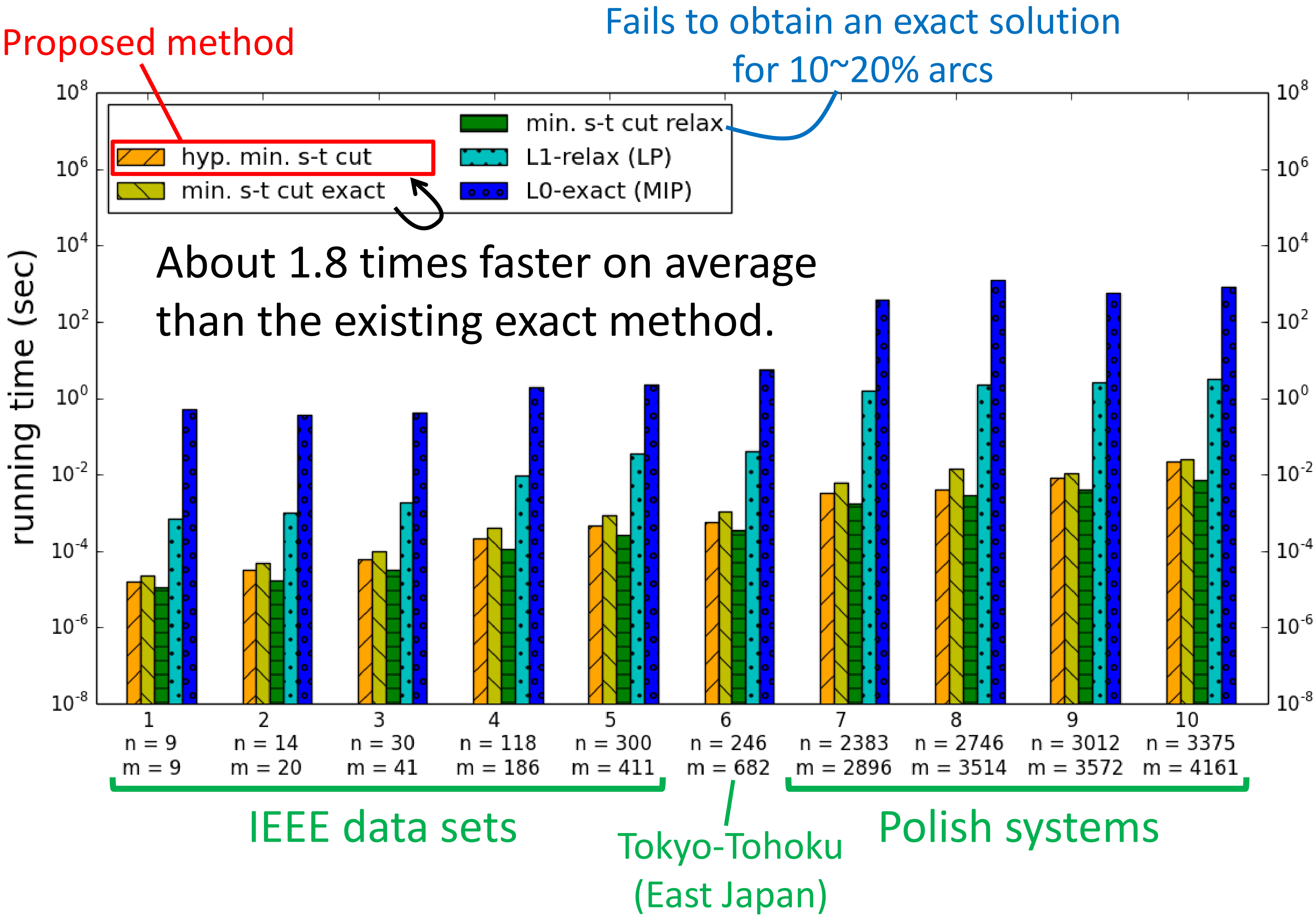
Problems and Solution Methods

- Finding a **sparsest attack** in the whole network
  - **hyp. global min. cut**: exact method by hypergraph min-cut
- Computing the **security index of an arc** $a \in A$
  - **hyp. min. s-t cut**: exact method by hypergraph min-cut
  - **min. s-t cut exact**: exact method by min-cut in auxiliary graph (Hendrickx et al. 2012)
  - **min. s-t cut relax**: approx. method by min-cut in input graph (Sou et al. 2011)
  - **L1-relax (LP)**: approx. method by LP-relaxation (Sou et al. 2013)
  - **L0-exact (MIP)**: exact method by MIP solver (CPLEX)

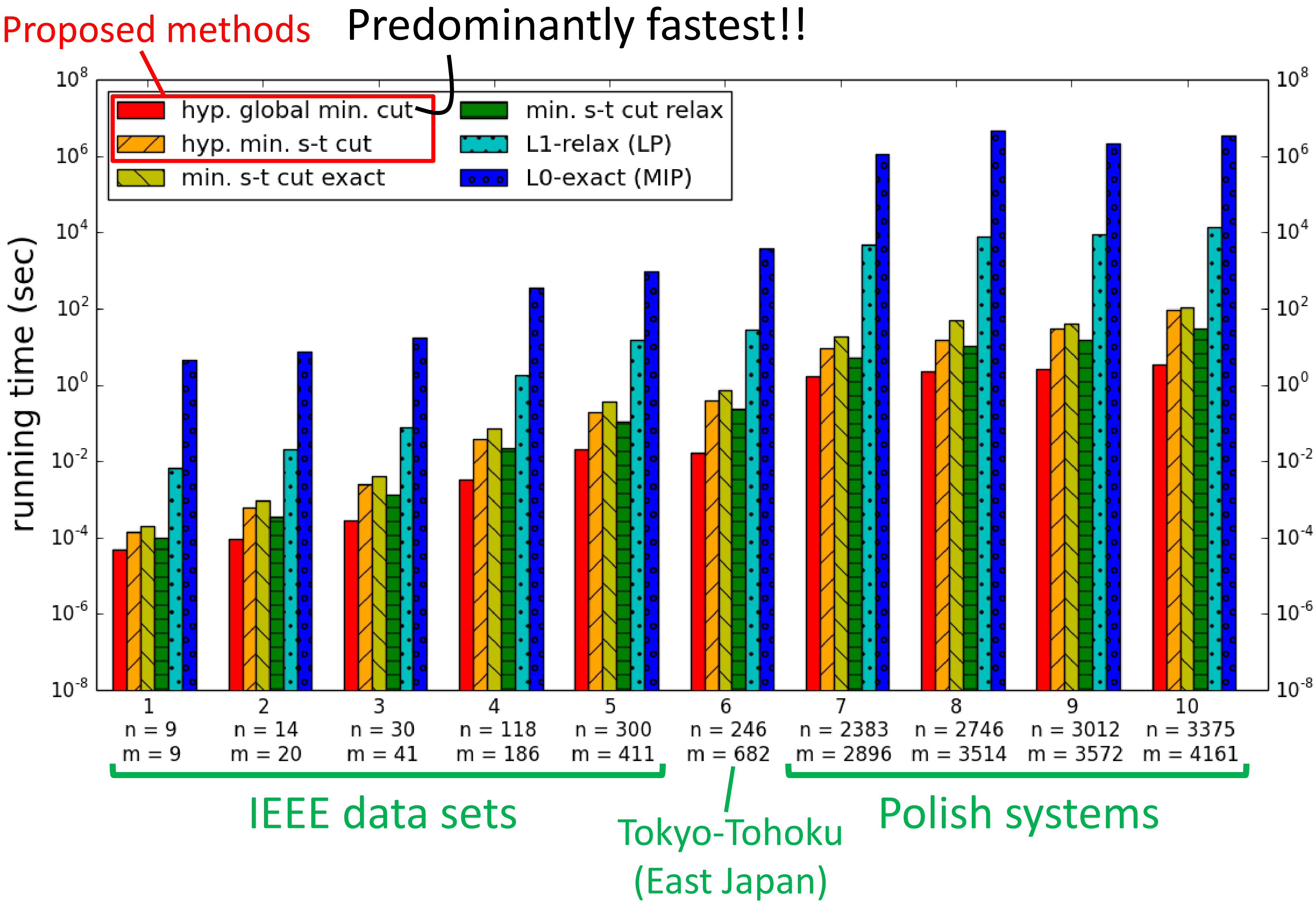
Proposed methods

$$\begin{aligned} \underset{\Delta\theta \in \mathbf{R}^V}{\text{minimize}} \quad & \|H\Delta\theta\|_0 \\ \text{subject to} \quad & H_a \Delta\theta \neq 0 \end{aligned}$$

Computational Time for **Security Index**



Computational Time for **Sparsest Attack**



Conclusion

- A **sparsest attack** and **the security index** of each measurement point are significant **security criteria for power networks**.
- A **sparsest attack** can be found **fast and exactly** by finding a **minimum cut in a hypergraph**.
- The **security index** of each measurement point can be computed **fast and exactly** by finding a **minimum s—t cut in a hypergraph**.